5.3 - Double-Angle Identities

Deriving the Double Angle Identities

$\sin(2A)$ and $\cos(2A)$

Start with the Sine sum identity

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A+A) = \sin A \cos A + \cos A \sin A$$

$$\boxed{\sin(2A) = 2 \sin A \cos A}$$

Start with the Cosine sum identity

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A+A) = \cos A \cos A - \sin A \sin A$$

$$\boxed{\cos(2A) = \cos^2 A - \sin^2 A}$$

5.3 - Double-Angle Identities

Deriving more Double Angle Identities

Start with the Cosine double angle identity

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = (1 - \sin^2 A) - \sin^2 A$$

$$\boxed{\cos(2A) = 1 - 2\sin^2 A}$$

Start with the Cosine double angle identity

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = \cos^2 A - (1 - \cos^2 A)$$

$$\boxed{\cos(2A) = 2\cos^2 A - 1}$$

5.3 - Double-Angle Identities

Deriving the Tangent Double Angle Identity

$$\boxed{\tan(2A) = \frac{\sin(2A)}{\cos(2A)}}$$

Or start with the Tangent sum identity

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$\boxed{\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}}$$

5.3 - Double-Angle Identities

If $\sin x = -\frac{4}{5}$ and $\cos x < 0$, find $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$.

Find $\cos x$

$$\sin^2 x + \cos^2 x = 1$$

$$\left(-\frac{4}{5}\right)^2 + \cos^2 x = 1$$

$$\cos^2 x = \frac{9}{25}$$

$$\cos x = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

$$\sin(2x) = 2\sin x \cos x$$

$$\sin(2x) = 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$$

$$\sin(2x) = \frac{24}{25}$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos(2x) = \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$$

$$\cos(2x) = -\frac{7}{25}$$

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)}$$

$$\tan(2x) = \frac{24/25}{-7/25}$$

$$\tan(2x) = -\frac{24}{7}$$

5.3 - Double-Angle Identities

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

Practice

Simplify

1. $\dfrac{1 - \cos 2x}{1 + \cos 2x}$

$\tan^2 x$

2. $\dfrac{1 + \sin x - \cos 2x}{\cos x + \sin 2x}$

$\tan x$

5.3 - Double-Angle Identities

Practice

If $\cos x = \frac{3}{5}$ and $\sin x < 0$, find $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$.

$$\sin(2x) = -\frac{24}{25}$$

$$\cos(2x) = -\frac{7}{25}$$

$$\tan(2x) = \frac{24}{7}$$

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

5.3 - Double-Angle Identities

$$\sin(2A) = 2\sin A\cos A \qquad \cos(2A) = 1 - 2\sin^2 A$$

$$\cos(2A) = \cos^2 A - \sin^2 A \qquad \cos(2A) = 2\cos^2 A - 1$$

Practice - Establish the identity

a. $(\sin x - \cos x)^2 = 1 - \sin(2x)$

b. $\cos(3x) = (1 - 4\sin^2 x)\cos x$

5.1 - Trigonometric Identities

Practice - Simplify

Hint: Convert $\tan x$ into $\sin x/\cos x$ and $\cot x$ into $\cos x/\sin x$

$$y = \frac{\cot x - \tan x}{\cot x + \tan x}$$

$$y = \cos(2x)$$

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